



# Infinitely Repeated Games: Utility

Game Theory Course: Jackson, Leyton-Brown & Shoham

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#### Definition

Given an infinite sequence of payoffs  $r_1, r_2, \ldots$  for player *i*, the average reward of *i* is

$$\lim_{k \to \infty} \sum_{j=1}^k \frac{r_j}{k}.$$

### **Discounted** reward

### Definition

Given an infinite sequence of payoffs  $r_1, r_2, \ldots$  for player *i* and discount factor  $\beta$  with  $0 < \beta < 1$ , *i*'s future discounted reward is

- $\overline{\sum_{j=1}^{\infty}}\beta^j r_j$
- Two equivalent interpretations of the discount factor:
  - 1. the agent cares more about his well-being in the near term than in the long term

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- $\sum_{j=1} \beta^j r_j.$
- Two equivalent interpretations of the discount factor:
  - 1. the agent cares more about his well-being in the near term than in the long term
  - 2. the agent cares about the future just as much as the present, but with probability  $1 \beta$  the game will end in any given round.

